SPATIAL FILTERING

Mechanism of SPATIAL FILTERING

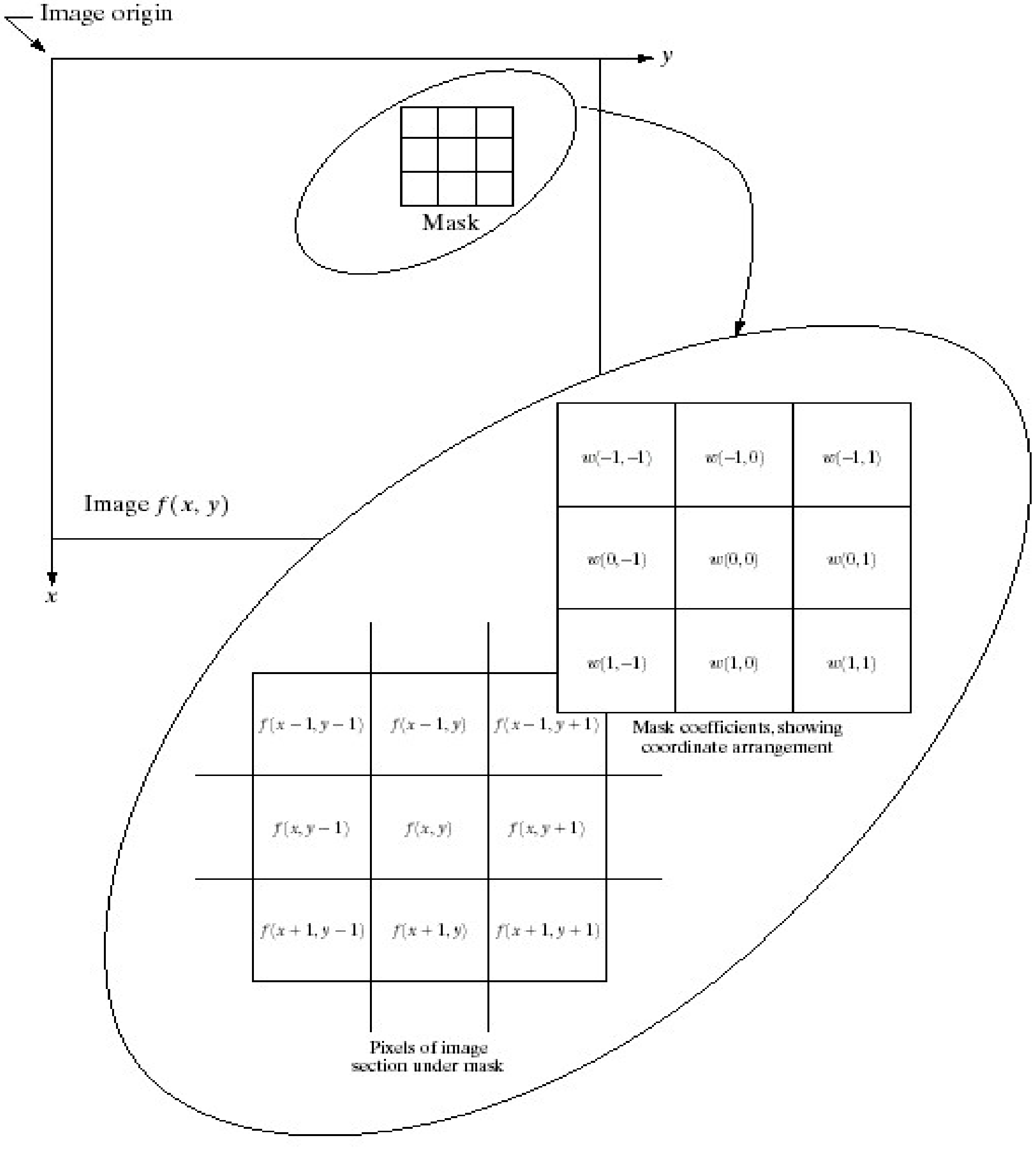


FIGURE 3.32 The mechanics of spatial filtering. The magnified drawing shows a 3 × 3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

Spatial Filtering

- Also called as masks processing

Two types: Linear and nonlinear filters

Linear Filters: LPF, HPF and BPF

- Low-pass filters eliminate or attenuate high frequency components in the frequency domain (sharp image details), and result in image blurring.
- High-pass filters attenuate or eliminate low-frequency components (resulting in sharpening edges and other sharp details).
- Band-pass filters remove selected frequency regions between low and high frequencies (for image restoration, not enhancement).

Spatial Filtering

In general, Linear filtering of an image f of size M X N with a filter mask of size m X n is given by the expression:

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

$a=(m-1)/2$ and $b=(n-1)/2$,

m x n (odd numbers)

- For $x=0,1,\ldots,M-1$ and $y=0,1,\ldots,N-1$
- Also called *convolution* (primarily in the frequency domain)

Spatial Filtering

The basic approach is to sum products between the mask coefficients and intensives of the pixels under the mask at a specific location in the image

The general 3 X 3 mask is shown in fig

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

The response of a linear mask is

$$R = w_1 z_1 + w_2 z_2 + \ldots\ldots + w_9 z_9$$

Where z_1 to z_9 --> gray levels of pixels

Spatial Filtering

Non Linear Filters:

These filters operate on neighborhoods.

Operation is based directly on the values of the pixels in the neighborhood under consideration

Noise reduction can be achieved effectively with a nonlinear filters.

SMOOTHING FILTERING

Smoothing Filters

Used for blurring and for noise reduction.

Blurring is used in preprocessing steps, such as removal of small details from an image prior to object extraction.

Noise reduction can be accomplished by blurring with a linear filter and also by nonlinear filtering

Linear Filter (LPF):

The shape of the impulse response needed to implement a LPF indicates that filter has to have all +ve coefficients.

Smoothing Filters

- Linear Filtering:

Linear Filtering is also referred as average filters.

Replace every pixels by the average of the gray levels in the filter mask

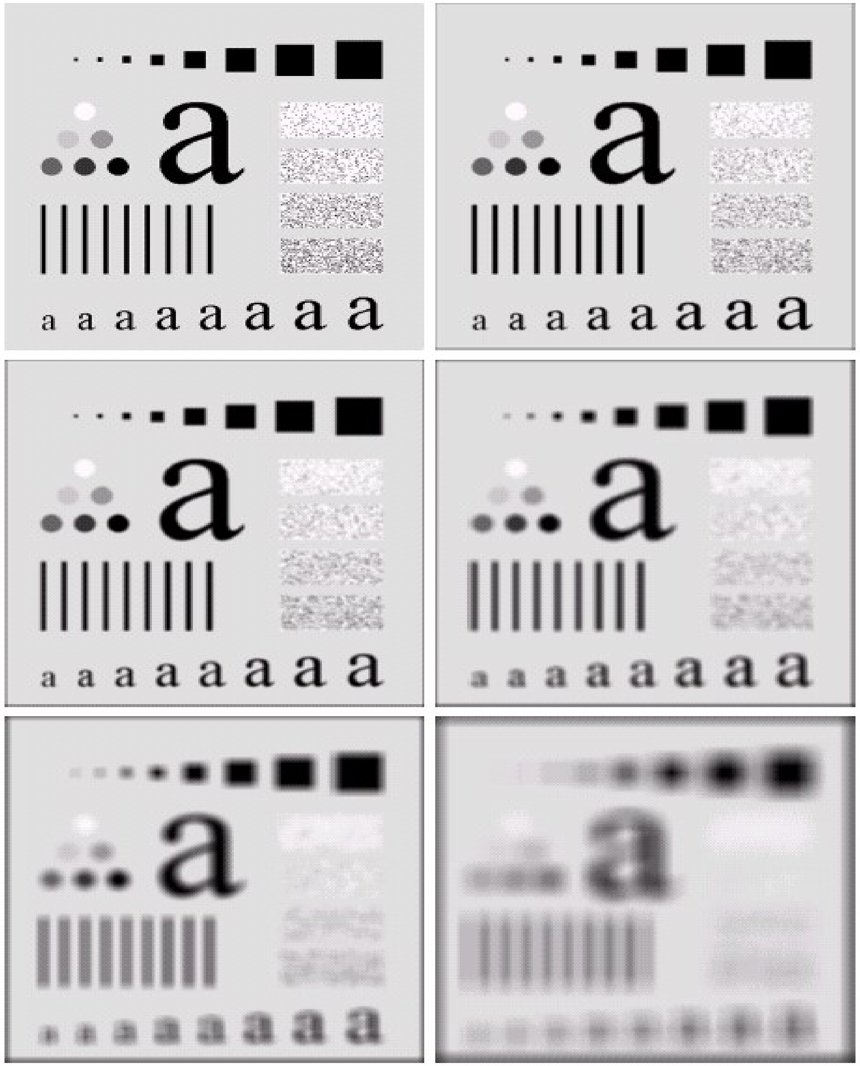
Reduces the sharp transitions in an image

Blur the edges of the image

Smoothing Filters

For a 3 X 3 spatial filter, the simplest arrangement would be a mask in which all coefficients has a value of 1.

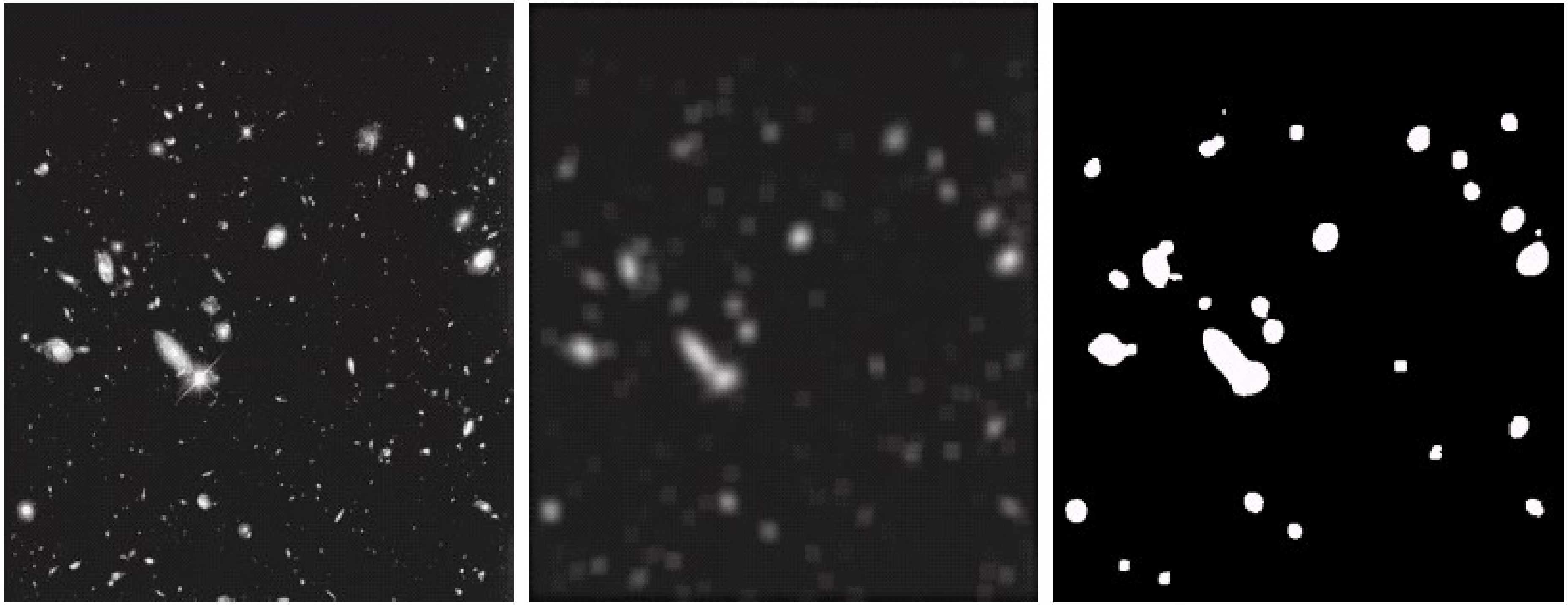
$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$



a b
c d
e f

FIGURE 3.35 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes n = 3, 5, 9, 15, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

Image Enhancement in the Spatial Domain



a b c

FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Smoothing Filters

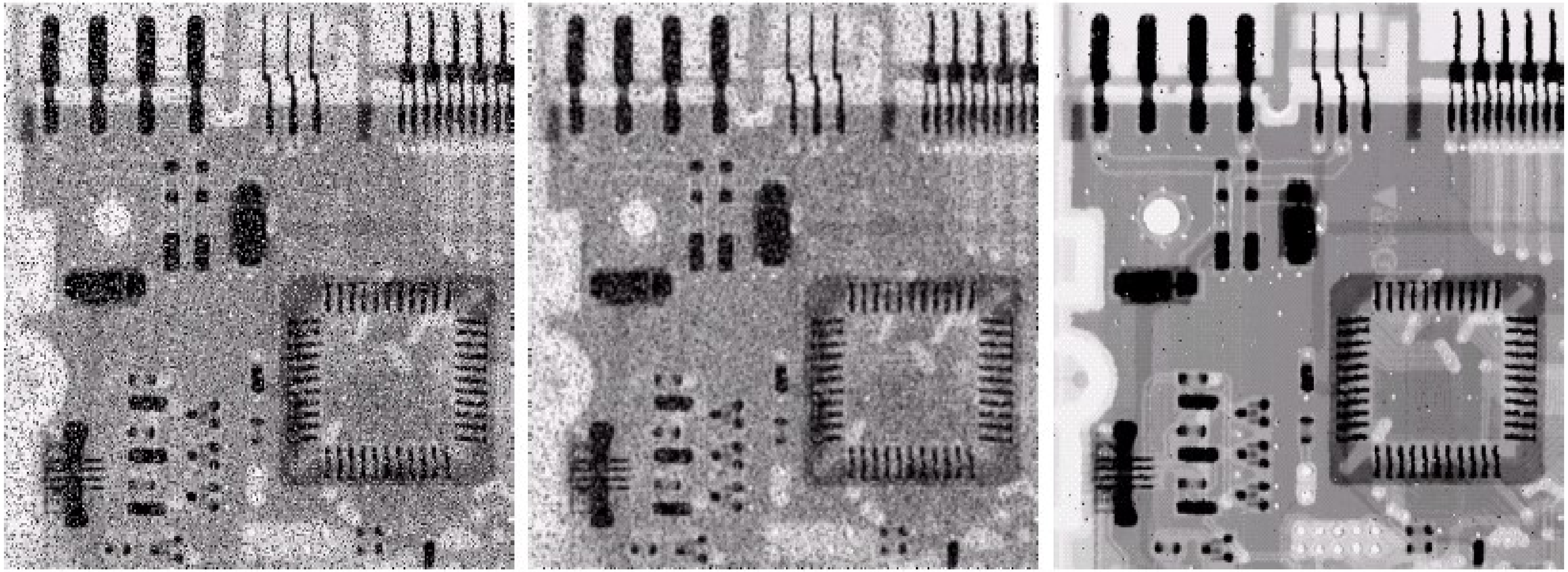
- Order Statistics Filters:

The difficulties of the Linear smoothening method is it blurs edges and other sharp details.

If the objective is noise reduction rather than blurring an alternative approach is to use median filter.

The gray levels of each pixel is replaced by median value of gray levels in a neighborhood of that pixel, instead of by averaging

This is more effective when noise pattern consists of strong isolated spikes



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

SHARPENING FILTERS

Sharpening Filters

- The Principal objective of sharpening filter is To highlight fine detail in an image or to enhance details that is blurred.
  - smoothing ~ integration
  - sharpening ~ differentiation

- Categories of sharpening filters:
  - Derivative operators
  - Basic highpass spatial filtering
  - High-boost filtering

Derivative Filters

- Averaging is analogous to integration and causes blurring
- Differentiation is expected to have opposite results and sharpen an image.
- The most common method of differentiation is the GRADIENT.
- First derivative $$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$
- Second derivative $$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Derivative Filters

- For a function f(x,y), the gradient of f at co-ordinates (x,y) is defined as the vector

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

- Consider the image as in the form of matrix, where the z's denote the values of gray levels

Z_1	Z_2	Z_3
Z_4	Z_5	Z_6
Z_7	Z_8	Z_9

Derivative Filters

- The image differentiation can be obtained by the magnitude of the vector ∇f
- $\nabla f \approx |z_5 - z_8| + |z_5 - z_6|$ *where* $z_5 - z_8$ *is in x direction*

  $z_5 - z_6$ *is in y direction*

Another approach for approximation is use of CROSS differentiation

- $\nabla f \approx |z_5 - z_9| + |z_6 - z_8|$

These two eq. can be implemented by using masks 2 x 2

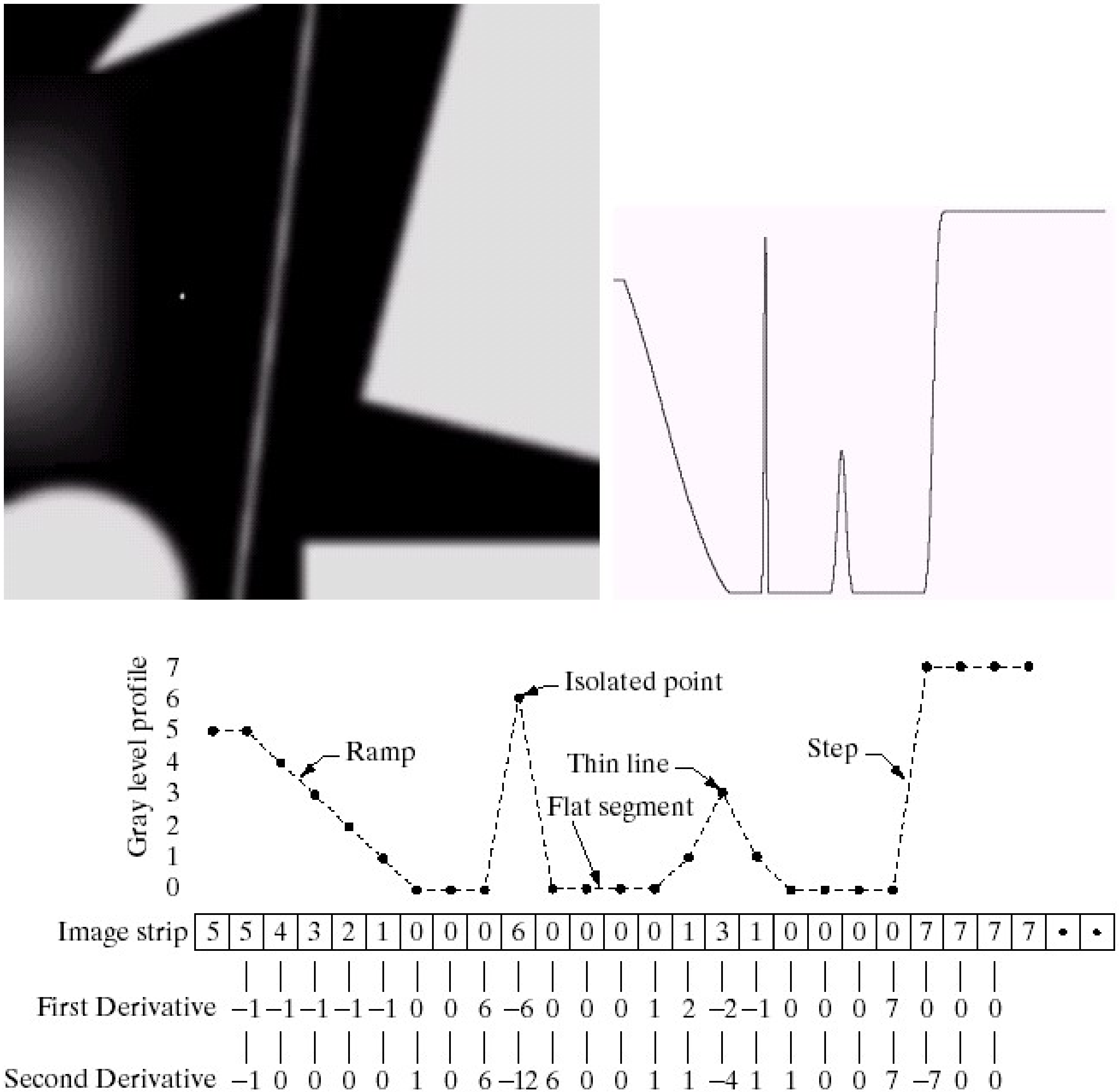
1	0
0	-1

0	1
-1	0

These two masks are called the ROBERT cross-gradient operators

a b
c

FIGURE 3.38 (a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point. (c) Simplified profile (the points are joined by dashed lines to simplify interpretation).



Digital Function Derivatives

- First derivative:
  - 0 in constant gray segments
  - Non-zero at the onset of steps or ramps
  - Non-zero along ramps

- Second derivative:
  - 0 in constant gray segments
  - Non-zero at the onset and end of steps or ramps
  - 0 along ramps of constant slope.

Observations

- 1st order derivatives produce thicker edges in an image
- 2nd order derivatives have stronger response to fine detail
- 1st order derivatives have stronger response to a gray lever step
- 2nd order derivatives produce a double response at step changes in gray level
- 2nd order derivatives have stronger response to a line than to a step and to a point than to a line

2-D, 2nd Order Derivatives for Image Enhancement

- Isotropic filters: rotation invariant
- Laplacian (linear operator):

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- Discrete version:

$$\frac{\partial^2 f}{\partial^2 x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial^2 y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

Laplacian

- Digital implementation:

$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] - 4f(x,y)$$

- Two definitions of Laplacian: one is the negative of the other
- Accordingly, to recover background features:

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & (I) \\ f(x,y) + \nabla^2 f(x,y) & (II) \end{cases}$$

I: if the center of the mask is negative

II: if the center of the mask is positive

Simplification

- Filter and recover original part in one step:

$$g(x,y)=f(x,y)-[f(x+1,y)+f(x-1,y)+f(x,y+1)+f(x,y-1)]+4f(x,y)$$

$$g(x,y)=5f(x,y)-[f(x+1,y)+f(x-1,y)+f(x,y+1)+f(x,y-1)]$$

- Laplacian increases the contrast of the image at the locations of gray level discontinuities.
- Laplacian restores overall gray levels
- Small details were enhanced and back ground tonanality perfectively preserved.

Image Enhancement in the Spatial Domain

(a)

0	1	0
1	−4	1
0	1	0

(b)

1	1	1
1	−8	1
1	1	1

(c)

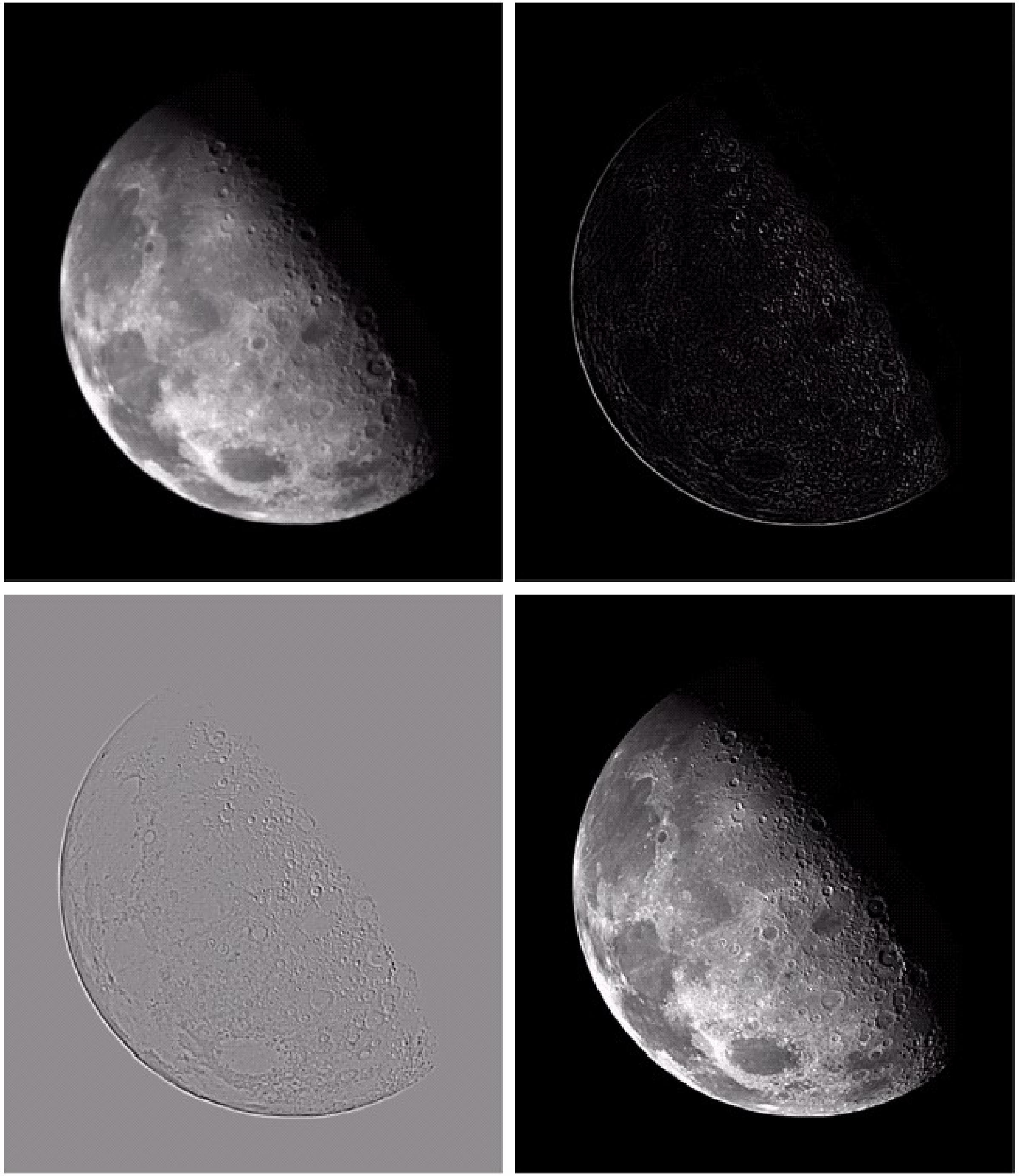
0	−1	0
−1	4	−1
0	−1	0

(d)

−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.39 (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.



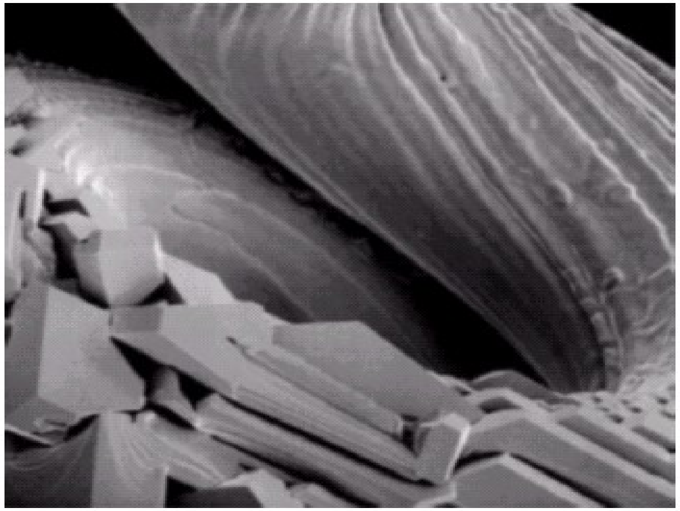
a b
c d

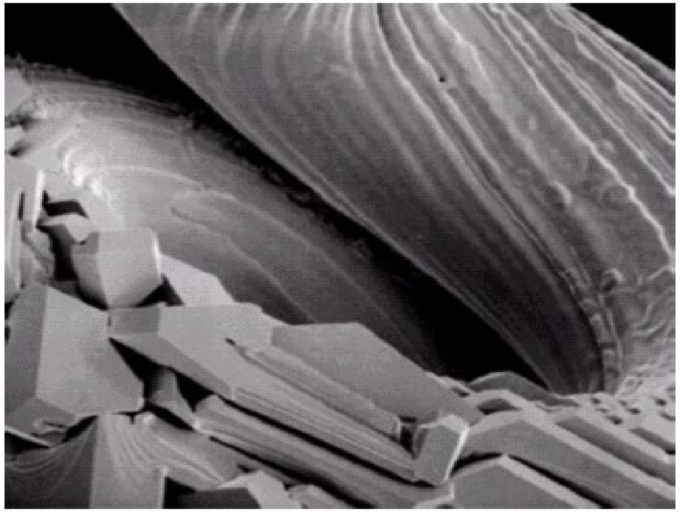
FIGURE 3.40 (a) Image of the North Pole of the moon. (b) Laplacian-filtered image. (c) Laplacian image scaled for display purposes. (d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)

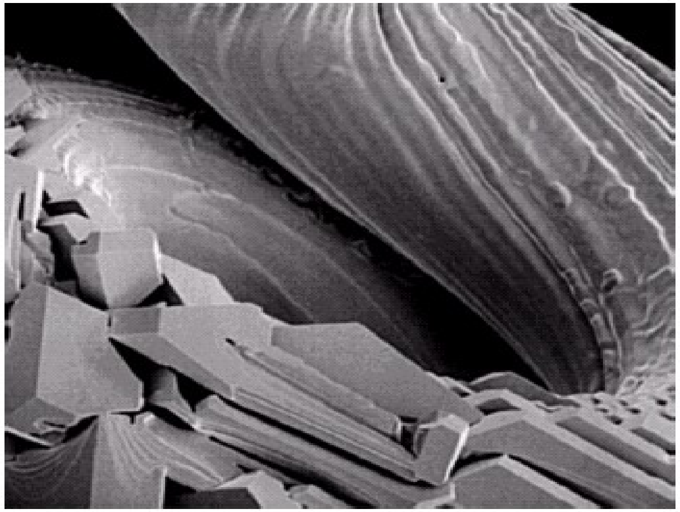
Image Enhancement in the Spatial Domain

0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1







a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Unsharp Masking

- Unsharp masking: $f_s(x,y) = f(x,y) - \overline{f}(x,y)$
- Highpass filtered image =

  Original – lowpass filtered image.

- If A is an amplification factor then:

  – High-boost = A · original – lowpass (blurred)

  = (A-1) · original + original – lowpass

  = (A-1) · original + highpass

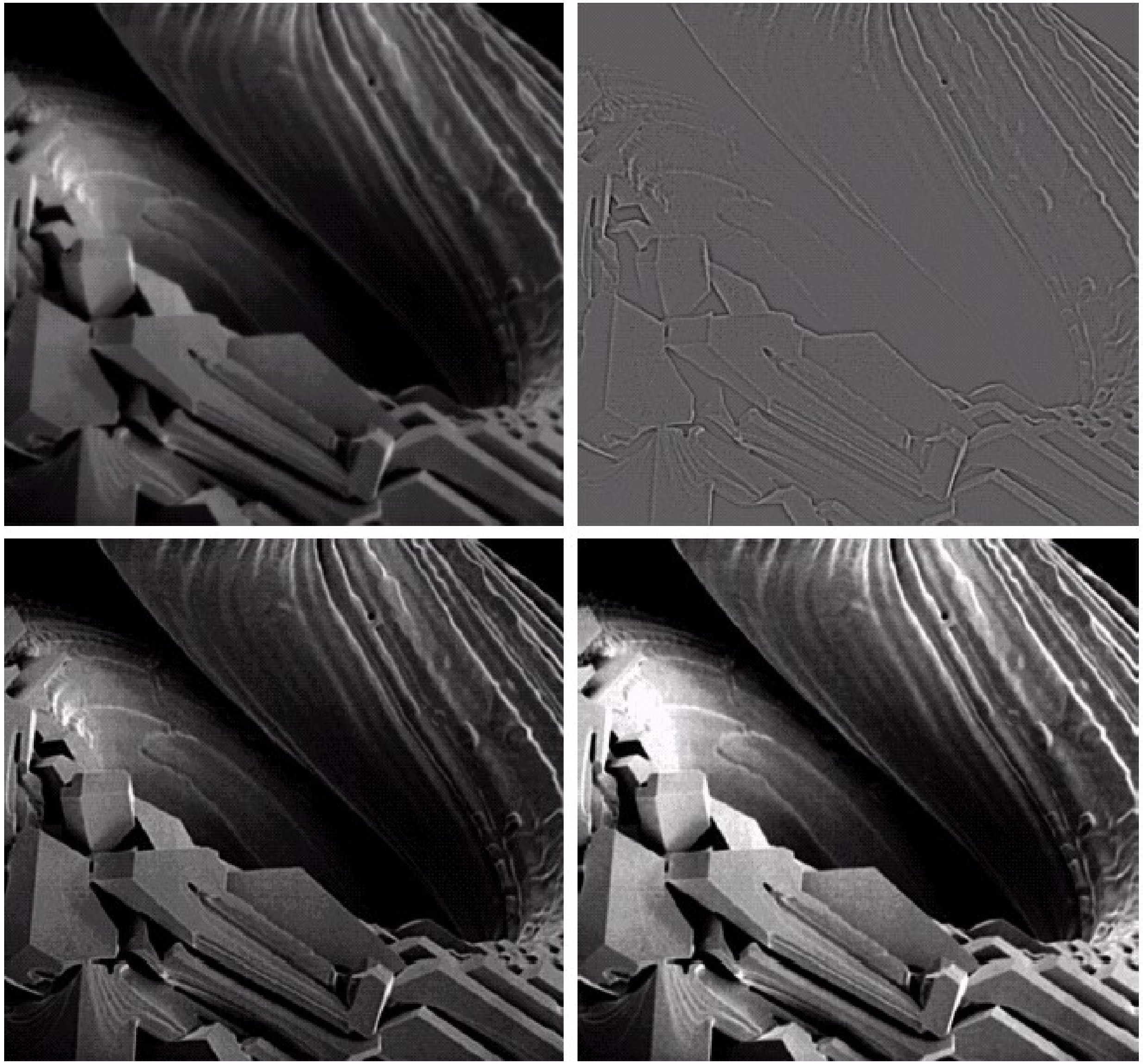
High-boost Filtering

- A=1 : standard highpass result
- A>1 : the high-boost image looks more like the original with a degree of edge enhancement, depending on the value of A.

$$\frac{1}{9} \times \begin{bmatrix} -1 & -1 & -1 \\ -1 & w & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

$w=9A-1, A \geq 1$

Image Enhancement in the Spatial Domain



a	b
c	d

FIGURE 3.43 (a) Same as Fig. 3.41(c), but darker. (a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$. (c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.

1st Derivatives

- The most common method of differentiation in Image Processing is the *gradient*:

$$\nabla F = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \quad \text{at } (x,y)$$

- The magnitude of this vector is:

$$\nabla f = mag(\nabla f) = [G_x^2 + G_y^2]^{\frac{1}{2}} = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

The Gradient

- Non-isotropic
- Its magnitude (often call the gradient) is rotation invariant
- Computations:

$$\nabla f \approx |G_x| + |G_y|$$

- Roberts uses:

$$G_x = (z_9 - z_5)$$

$$G_y = (z_8 - z_6)$$

- Approximation (Roberts Cross-Gradient Operators):

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

a
b c
d e

FIGURE 3.44 A 3×3 region of an image (the z's are gray-level values) and masks used to compute the gradient at point labeled z_5. All masks coefficients sum to zero, as expected of a derivative operator.

(a)

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

(b)

−1	0
0	1

(c)

0	−1
1	0

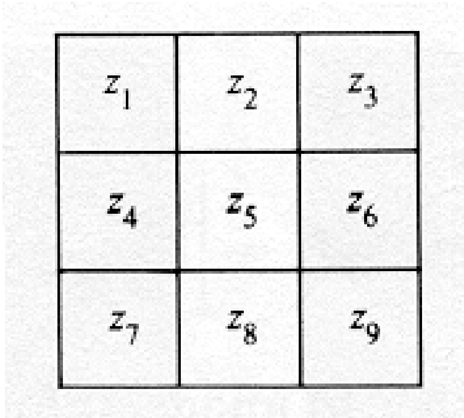
(d)

−1	−2	−1
0	0	0
1	2	1

(e)

−1	0	1
−2	0	2
−1	0	1

Derivative Filters



At z_5, the magnitude can be approximated as:

$$\nabla f \approx [(z_5 - z_8)^2 + (z_5 - z_6)^2]^{1/2}$$

$$\nabla f \approx | z_5 - z_8 | + | z_5 - z_6 |$$

Derivative Filters

- Another approach is:

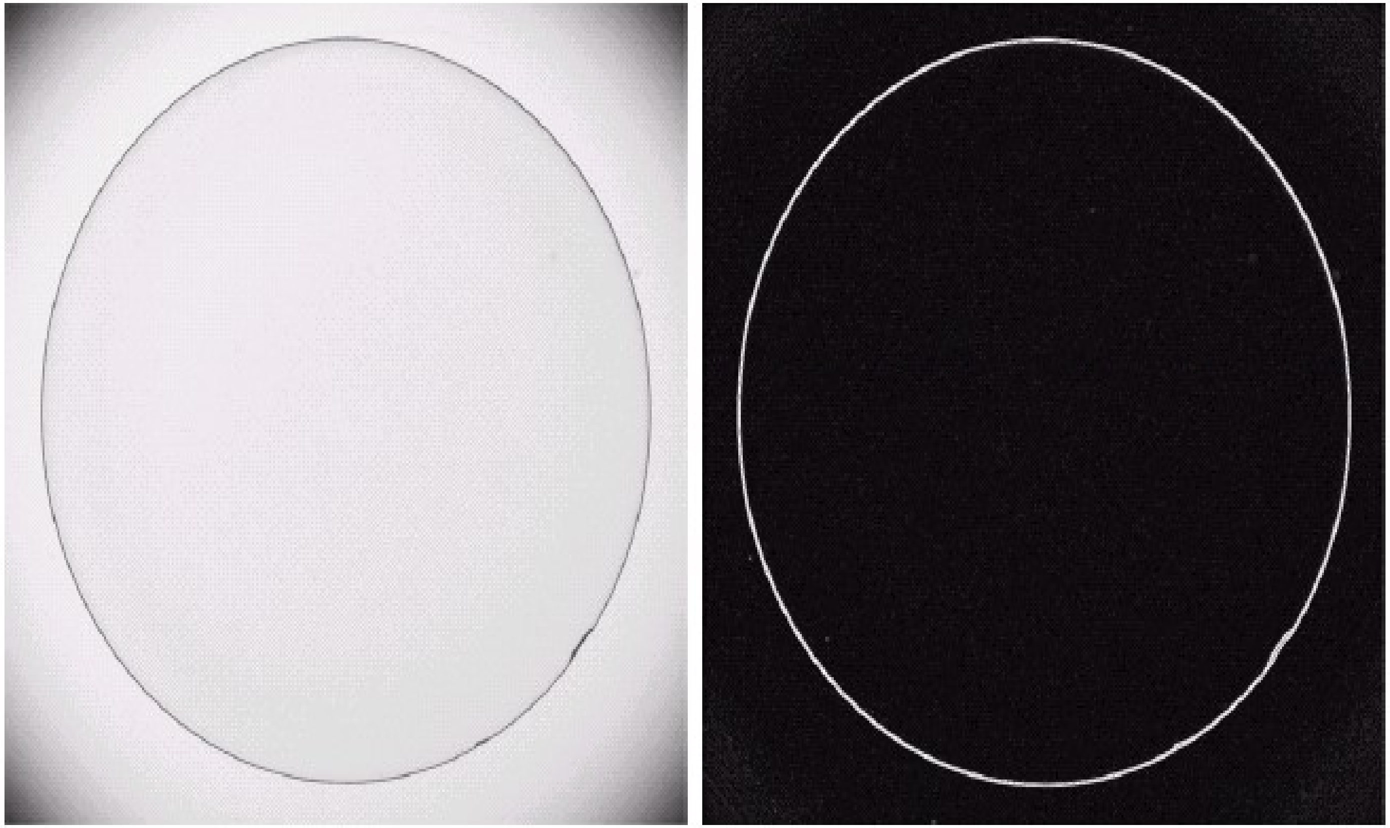
$$\nabla f \approx [(z_5 - z_9)^2 + (z_6 - z_8)^2]^{1/2}$$

$$\nabla f \approx | z_5 - z_9 | + | z_6 - z_8 |$$

- One last approach is (Sobel Operators):

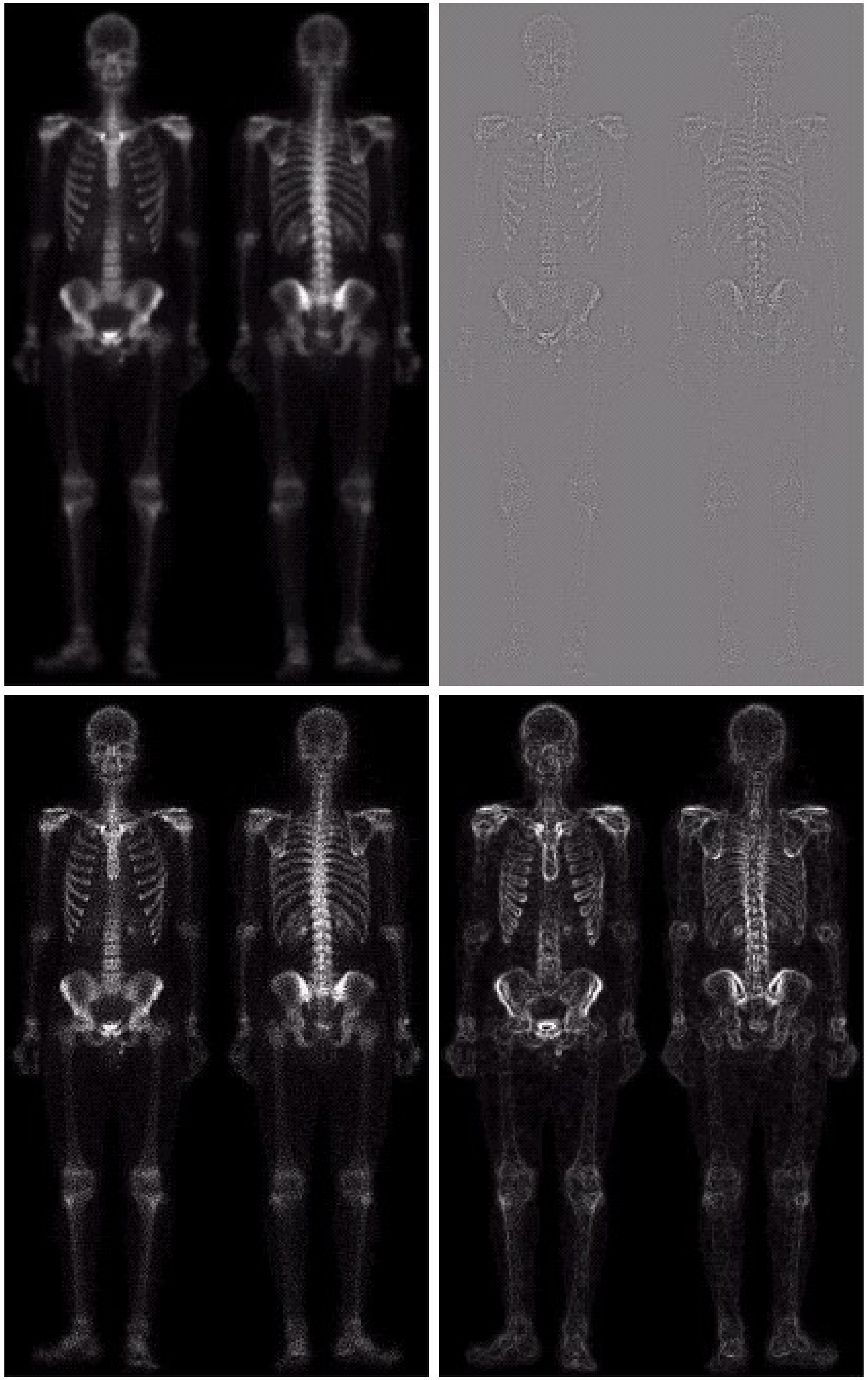
$$\nabla f = \left|(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)\right| + \left|(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)\right|$$

Image Enhancement in the Spatial Domain



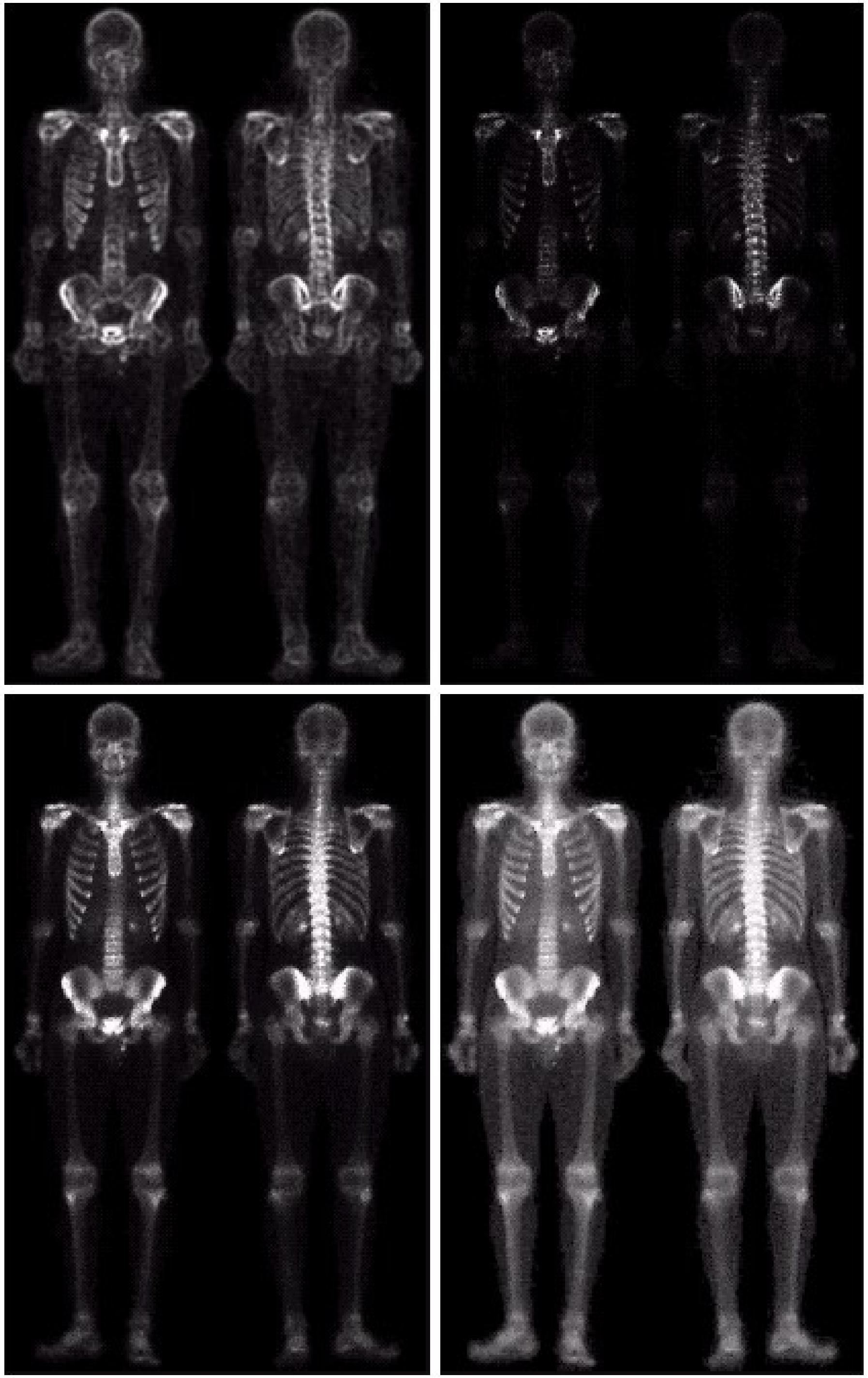
a b

FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)



a	b
c	d

FIGURE 3.46 (a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a).



e	f
g	h

FIGURE 3.46 *(Continued)* (e) Sobel image smoothed with a 5 × 5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)